Comment on "The Bright Side of Dark Matter"

In a significant recent paper A. Edery [1] undertakes a new study of light deflection in generalizations of general relativity (GR). He claims to prove that any metric-based gravitational theory that proposes to explain the flat rotation curves of disk galaxies without postulating dark matter halos must conflict with observations of gravitational lensing by galaxies and clusters of galaxies because any such theory inevitably make a negative contribution to light deflection. Here we show that some of the basic steps of Edery's argument are invalid, and no such general result obtains.

Edery considers the metric of a spherical mass M to be given by $ds^2 = B(M,r)dt^2 - A(M,r)dr^2 - r^2d\Omega^2$, with which he calculates the deflection of a light ray approaching from infinity and then receding to infinity. The agreement with the solar-system tests of general relativity requires that $AB \approx 1$ to high accuracy in the vicinity of the sun. Unjustifiably, Edery extends this requirement to galactic scale, where in conjunction with the requirement of flat rotation curves, it leads to suppressed light deflection. (It is easy to see that without the constraint $AB = 1$, flat rotation curves and enhanced light bending are consistent.) But solar–system tests do not constrain the form of A or B on galactic scale (except in the context of a specific gravitation theory). Consider, for example, a modified-gravity theory with a mass scale, M_0, below which it merges with GR [i.e. $A^{-1}(M,r) = B(M,r) = 1-2GM/r$ for $M < M_0$]. If M_0 is between a solar mass and galactic masses, all solar-system results agree with those of GR, while the form of A or B on galactic scale depends on the exact theory. Similarly, if the departure from GR occurs only above a certain length scale, intermediate between interstellar and galactic scale, $AB = 1$ in the solar–system, there is clearly no anomalous contribution to light bending within the solar system, while again nothing can be generically deduced about A or B on galactic scale.

In this connection we note another deficiency in Edery's arguments: he calculates the deflection angle accumulated along the ray's path all the way from infinity, and much of the undesirable negative contribution comes from the asymptotic region. However, Edery's assumed form of the metric is only valid out to limited radii: for galaxies only to a few megaparsecs where the growth of the gravitational potential saturates as the galaxy's field merges with the cosmological one; and near the sun only to a tenth of a parsec where the mean field of the galaxy takes over. Alternative gravity theories are generically nonlinear, so one cannot consider the contribution of the sun separately from its galactic environment. Edery has also failed to realize that in solar light–deflection experiments, only the *difference* of deflection angles for two light paths, one grazing the sun, and one passing about one earth–sun distance away, is actually measured. In such difference the contribution from large distances, so crucial to his point about negative light deflection, tends to cancel out. Therefore, it is easy to devise phenomenologically valid theories in which solar–system and galactic predictions are unconnected. Edery supposes such connection as unavoidable because he fails to realize that (i) the field of a totally isolated mass is phenomenologically relevant only up to a limited distance, and (ii) the sun and galaxies are sufficiently different in mass, size, etc., so as to permit theories that describe the two cases with totally different metric coefficients. This occurs, for example, in theories in which, in the spirit of MOND [2], departure from standard gravitation sets in only below a certain acceleration scale (which is of order of the sun's acceleration at a fraction of the interstellar distance).

If an alternative theory exhibits $AB \neq 1$ on galactic scale, no contradiction between flat rotation curves and enhanced light–bending need appear. Both desired features coexist in Sanders' stratified theory [3], which also predicts the PPN parameters for solar system tests in the measured ranges. We also note that the dark-matter plus GR standard doctrine eludes Edery's argument for suppressed deflection by having $AB \neq 1$ around galaxies (since the matter density even outside the visible galaxy does not vanish), but $AB \approx 1$ near the sun where no dark matter is needed. But Edery does not tell us why we cannot have a modified–gravity theory that gives for the metric of the visible matter in the whole universe exactly what GR gives with dark matter ? Indeed, his statements are not really about theories, but about metrics coming from unspecified equations. By claiming that he only assumes a metric theory (one that obeys the equivalence principle), he is driven to the conclusion that there is no metric for the world that gives at the same time flat rotation curves, enhanced light bending by galaxies, and consistency with the solar–system tests. This would seem to exclude the metric calculated from GR with dark matter - a symptom of the untenability of Edery's sweeping claim.

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